IR safety & jet cross sections

We have seen that for the total cross section, the soft and collinear divergences cancel in the final result. An observable for which this happens is called IR safe. It is interesting to ask which conditions an observable must fullfill to be IR safe. Observables depend on the kinematic configuration, i.e. the observable will have a value O_2 on the fixed two parton phase space and a value $O_3(y_1, y_2)$ on a given three-particle phase space point:

virtual corrections

$$\langle O \rangle = \sigma^{(0)}_{q\bar{q}} \cdot \Big\{ O_2 (1 + V)$$

phase space $* |m|^2$

$$\int_0^1 dy_1 \int_0^{1-y_1} dy_2 \, R(y_1, y_2) \cdot O_3(y_1, y_2) \Big\}$$

To maintain the cancellations of IR divergences, we must choose observables for which $O_3 \to O_2$ in the singular limits $y_1 \to 0$ and $y_2 \to 0$.

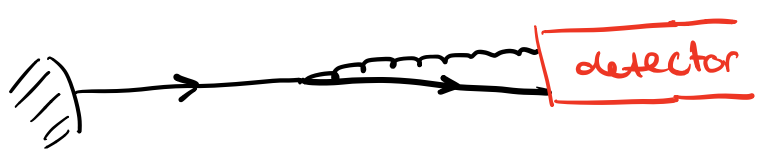
In more physical terms, the observable with a collinear or soft parton, must be equal to the observable with one less parton. More generally

$$\mathcal{O}_{n+1}(p_1, \dots, p_n, k) \xrightarrow{k \to 0} \mathcal{O}_n(p_1, \dots, p_n)$$

$$\mathcal{O}_{n+1}(p_1, \dots, p_n, p_{n+1}) \to \mathcal{O}_n(p_1, \dots, p_n + p_{n+1})$$

(exercises)

This makes physical sense: if a particle is extremely soft, it is unobservable. The same is true if we consider two massless collinear particles



The IR divergences from soft emission are also present in QED and if we set $m_f = 0$ also the collinear ones.

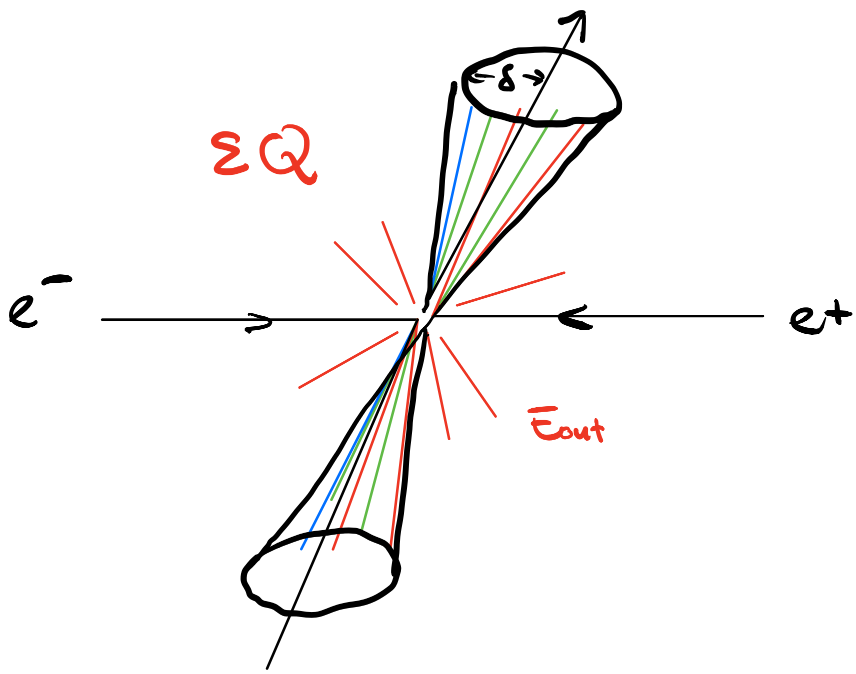
Q: is $\sigma(e^+e^- \to e^+e^-)$ IR safe?

The soft and collinear phase-space regions (and loop integration regions) are associated with low-energy physics. That these contributions drop out is a necessary condition that an observable can be computed in perturbation theory.

The analysis of these low energy regions can be done using an effective-theory framework: Soft-collinear effective theory (SCET). This will be discussed later in the lecture.

Jet observables

Note that the process $\sigma(e^+e^- \to q\bar{q})$ is <u>not</u> IR safe. This would correspond to $O_2 = 1$ and $O_3 = 0$, which would spoil the cancellation of IR singularities. However, it would nice to define an IR safe observable that distinguishes $q\bar{q}$ from $q\bar{q}g$. To be infrared safe, we need to include soft & collinear radiation together with the particles produced in the initial hard reaction. The first such definition is due to Sterman & Weinberg.



They consider two oppositely directed cones with opening half-angle δ.

An event contributes to the 2-jet jet cross section σ if at most a fraction εQ of the energy is outside the jet cones.

Let us quickly check that this cross section is IR safe at NLO:

1.) The virtual corrections to $\sigma_{q\bar{q}}$ fully contribute

2.) If two particles are colliner for $q\bar{q}g$, then they are always inside the cone. If a particle becomes very soft it is always included.

$\Rightarrow$ IR divergences cancel between real and virtual.

Sterman & Weinberg computed $\sigma(\varepsilon, \delta)$ and found

collinear log   soft log.

$$\sigma(\varepsilon, \delta) = \sigma^{(0)}_{q\bar{q}} \left(1 + \frac{\alpha_s C_F}{4\pi}\left(4 \ln\delta \ln 2\varepsilon + 3\ln\delta + \pi^2/3 - 5/2\right)\right)$$

for $\varepsilon \ll 1$ & $\delta \ll 1$. Corrections enhanced by logs!

The Sterman - Weinberg cross section is suitable for events with <u>two</u> jets, but it is of course possible to generalize the construction to more jets by working with n cones.

A definition of a multi-jet cone cross section needs

A.) Prescription to select the cone directions.

B.) A "merge-split" prescription to deal with overlapping cones.

It turns out that it is nontrivial to implement A.) & B.) in an infrared safe way.

The directions of the cones must point along the direction of the total momentum of the particles inside the cone. One then chooses the cone with the most energy inside, checks if other cones are overlapping, splits the energy according to B.) and repeats.

To find all stable cones, one should consider all subsets of particles, but their number $\sim 2^N$ increases rapidly, so experimentalists worked with shortcuts, which typically spoilt IR safety.

An N^2 efficient algorithm is obtained by considering pairs of particles lying on the edge of a cone, see slides and 0704.292 by Salam & Soyez and 0906.1833 by G. Salam.

Sequential clustering in e^+e^-

Most modern measurements do not use cones, but work with jet definitions that cluster particles in momentum space, based on some distance measure. The simplest possibility at an e^+e^- collider is the JADE algorithm:

1.) For all pairs of particles, compute:

$$d_{ij} = \frac{2E_iE_j(1-\cos\theta_{ij})}{Q^2} = \frac{(p_i-p_j)^2}{Q^2}$$

for $p_i^2 = p_j^2 = 0$

2.) Find minimum d_{min} of the d_{ij}'s.

3.) If $y_{min} < y_{cut}$, merge i and j into new particle with momentum $p = p_i + p_j$. Go to 1.)

4.) Declare the remaining particles jets and terminate.

The smaller d_{cut}, the more jets. The algorithm is IR safe because soft & collinear particles are immediately clustered.

The JADE algorithm has the disadvantage that it clusters soft particles even if they go in opposite directions. To remedy this, one can modify d_{ij} to

$$d_{ij} = \frac{2\min(E_i^2, E_j^2)(1-\cos\theta_{ij})}{Q^2}$$

"k_T algorithm for e^+e^-"

With this measure, the soft radiation gets clustered into particles along similar directions.

Sequential clustering at hadron colliders

The above algorithms are not directly suitable for the LHC:

1.) Collisions do not take place in CMS, Q^2 unknown.

2.) There is always significant radiation along the beam directions (proton remnant).

To deal with 1.) one typically works at hadron colliders with the variables

1.) $p_T = \sqrt{p_x^2 + p_y^2}$ momentum transverse to beam along z-axis

2.) $\phi = \arctg p_y/p_x$ azimuthal angle

3.) $y = \frac{1}{2} \ln\left(\frac{E+p_z}{E-p_z}\right)$ rapidity

1.) & 2.) are invariant under boosts along the beam direction. The rapidity transforms as

$$\begin{pmatrix} E' \\ p_z' \end{pmatrix} = \begin{pmatrix} \cosh(\beta) & -\sinh(\beta) \\ -\sinh(\beta) & \cosh(\beta) \end{pmatrix} \begin{pmatrix} E \\ p_z \end{pmatrix}$$

and $y' = y - \beta$

Rapidity differences are invariant!

This motivates the following distance measure

$$d_{ij} = \min\left(p_{T_i}^{2p}, p_{T_j}^{2p}\right) \frac{\Delta R_{ij}^2}{R^2}$$

"jet radius" parameter

$$d_{iB} = p_{T_i}^{2p}$$

"distance to beam"

with

$$\Delta R_{ij}^2 = (y_i - y_j)^2 + (\phi_i - \phi_j)^2$$

$p = 1$: k_T - algorithm (Catani et al., Ellis & Soper '93)

$p = 0$: C/A algorithm (Wobisch & Wengler '99)

$p = -1$: anti - k_T (Cacciari, Salam & Soyez '08)

Clustering sequence:

1.) Compute d_{ij}'s, d_{iB}'s. Find minimum

2.) If d_{ij} is minimum, combine i & j, goto 1.)

3.) If d_{iB} is minimum, remove i from list, declare it a jet, goto 1.)

4.) Stop when no particles remain.

Almost all LHC measurements use anti-k_T with $R \sim 0{,}5$. Anti-k_T produces very cone-like jets because it first clusters the energetic partons and absorbs the soft radiation at the end, see Figures.